UNIVERSAL SIMULATION OF DEGENERATING HOMOGENEOUS TURBULENCE

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The problem of universal simulation of the dynamics of a turbulent velocity field (universal in the sense of arbitrary values of the Reynolds turbulence number) is treated on the basis of the moment model in the second approximation.

One of the modern methods in the semiempirical theory of turbulent shear flow is based on the use of differential equations for the single-point statistical moments of hydrodynamic fields. This mathematical apparatus has evolved from the need to simulate the exact equations for those moments, for the purpose of their closure. The simulating equations must satisfy several necessary conditions, one of them being that they correspond to exact laws of field dynamics the simplest (homogeneous and isotropic) form.

The dynamics of a homogeneous velocity field can be described by a pair of equations, for the turbulence energy and for the vorticity

$$\begin{cases} \langle \dot{q^2} \rangle + 2\,\varepsilon_u = 0, \\ \dot{\varepsilon}_u + F_u(N_{Re,\lambda})\,\varepsilon_u^2 / \langle q^2 \rangle = 0. \end{cases} \quad (1)$$

The function $F_u(N_{Re,\lambda})$ in the isotropic case is

$$\begin{gathered} F_u(N_{Re,\lambda}) = 7\,(S_u + S_\nu)\,N_{Re,\lambda}/15\sqrt{3}, \\ S_u = \langle (\partial u_r/\partial x_r)^3 \rangle / \langle (\partial u_r/\partial x_r)^2 \rangle^{3/2}, \\ S_\nu = 2\,\nu \langle (\partial^2 u_r/\partial x_r^2)^2 \rangle / \langle (\partial u_r/\partial x_r)^2 \rangle^{3/2}. \end{gathered} \quad (2)$$

The dimensionless moments S_u and S_ν characterize, respectively, the increase of the vorticity of the velocity fluctuation field due to stretching of vortices ($S_u < 0$) and the decrease of ε_u due to viscosity ($S_\nu < 0$).

The degeneracy of homogeneous and isotropic turbulence can be adequately described with the aid of the second-order moment model (1), if only the function $F_u(N_{Re,\lambda})$ is given for all possible values of the Reynolds number. In order to determine the function $F_u(N_{Re,\lambda})$, it is necessary to know the dynamics of the third-order two-point moment $\langle u^2{}_r u'{}_r \rangle$, which characterizes the interaction of turbulent vortices of various scales (for small but finite values of $N_{Re,\lambda}$ this problem has already been solved [1]). Determining $F_u(N_{Re,\lambda})$ for arbitrary values of $N_{Re,\lambda}$ is a problem equivalent to the well-known problem of the function $T(k, \tau)$ which characterizes the transfer of fluctuation energy over the frequency spectrum and which appears in the equation of energy spectrum dynamics

$$(\partial/\partial\tau + 2\,\nu k^2)\,E(k, \tau) = T(k, \tau), \quad (3)$$

being, moreover, related to S_u through the equality

$$S_u = -(30\sqrt{3}/14)\int_0^\infty T(k, \tau)\,k^2 dk \Big/ \Big[\int_0^\infty E(k, \tau)\,k^2 dk\Big]^{3/2}.$$

Several attempts have been made [2-10] to determine the function $F_u(N_{Re,\lambda})$ empirically or semiempirically from data on the degeneracy of homogeneous turbulence, and, as a result, semiempirical expressions have been proposed for $F_u(N_{Re,\lambda})$ in the form of functions monotonically varying over the range bounded by the two asymptotes $N_{Re,\lambda} \to 0$ and $N_{Re,\lambda} \to \infty$. The asymptotic values $F_u(0)$ and $F_u(\infty)$ can be found from all well-known limiting invariance relations describing the degeneracy of a homogeneous and isotropic velocity field. Indeed, assuming

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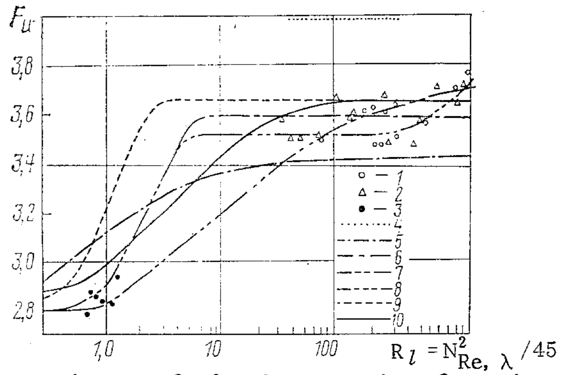

Fig. 1. Dependence of the interaction function F_u on the Reynolds turbulence number $N_{Re,\lambda}$ according to data: 1) [13]; 2) [14]; 3) [15]; 4) [19]; 5) this study; 6) [7]; 7) [10]; 8) [9]; 9) [8]; 10) approximation (8).

F_u = const along the asymptotes, one can put the solution to the system of equations (1) in the form

$$\begin{cases} \langle q^2 \rangle \varepsilon_u^{-2/F_u} = c_{1u}, \\ \langle q^2 \rangle = c_{1u}^{F_u/F_u-2} (F_u - 2)^{-2/F_u-2} (\tau + \tau_0)^{-2/F_u-2}, \end{cases} \tag{4}$$

where c_{1u} and τ_0 are constants, $\tau_0 = \langle q_0^2 \rangle / (F_u - 2)\varepsilon_{u0}$ being the virtual origin of degeneracy of the velocity field according to the law (4).

Using the Loitsyanskii invariance relation [11] (for $N_{Re,\lambda} \ll 1$)$\langle q^2 \rangle \lambda_u^5$ = const and the Saffman invariance relation [12] (for $N_{Re,\lambda} \gg 1$)$\langle q^2 \rangle L_u^3$ = const (where $L_u = 5\langle q^2 \rangle^{3/2}/\varepsilon_u$ is the scale of vortices with energy content), we obtain the asymptotic values of F_u

$$\lim_{N_{Re,\lambda} \to 0} F_u = 14/5, \quad \lim_{N_{Re,\lambda} \to \infty} F_u = 11/3. \tag{5}$$

It follows from the second of expressions (4) that, with F_u = const, the degeneracy of $\langle q^2 \rangle$ can be represented as a power-law relation

$$\langle q^2 \rangle = A \cdot (\tau + \tau_0)^{-n}, \tag{6}$$

i.e., F_u can be calculated as $F_u = 2(n + 1)/n$ with n having been determined by experiment. According to expressions (5), moreover, we have the asymptotic estimates

$$\lim_{N_{Re,\lambda} \to 0} n = 5/2, \quad \lim_{N_{Re,\lambda} \to \infty} n = 6/5.$$

Experimental data [13, 14] for large values of the Reynolds turbulence number ($N_{Re,\lambda} >$ 40) processed according to relation (6) indicate (Fig. 1) that the power exponent n is a constant ($n = 1.3 \pm 0.15$) which corresponds to the value of the interaction function $F_u \simeq 3.6$, which also agrees with the second asymptotic estimate, while other data [15] for small values of the Reynolds turbulence number ($N_{Re,\lambda} < 10$) confirm the asymptotic estimate (5) for weak turbulence. Note should be taken of the insufficiency of experimental data* for the $10 \lesssim N_{Re,\lambda} \leqslant 30$ range, where function F_u varies between its asymptotic values (5).

Because of the lack of reliable data on the degeneracy of turbulence in the intermediate range of $N_{Re,\lambda}$, great importance is attached to recently developing methods of numerical simulation applicable to degeneracy of homogeneous turbulence, particularly high ranking among them being the method of direct numerical integration of three-dimensional transient Navier–Stokes equations [16]. Less versatile are spectral function $E(k, \tau)$, with closure through various kinds of hypotheses of phenomenological or statistical nature. Phenomenological hypotheses (Obukhov's, Heisenberg's, Karman's, etc. [17]) contain empirical constants, which must be determined through comparison of the results of numerical solution with results of

*The data in [15] for the $10 \leqslant N_{Re,\lambda} \leqslant 16$ range are questionable, inasmuch as they yield values for F_u which greatly exceed the asymptotic estimate (5) for strong turbulence and thus yield a nonmonotonic (peaking) function $F_u(N_{Re,\lambda})$ in the intermediate range of $N_{Re,\lambda}$.

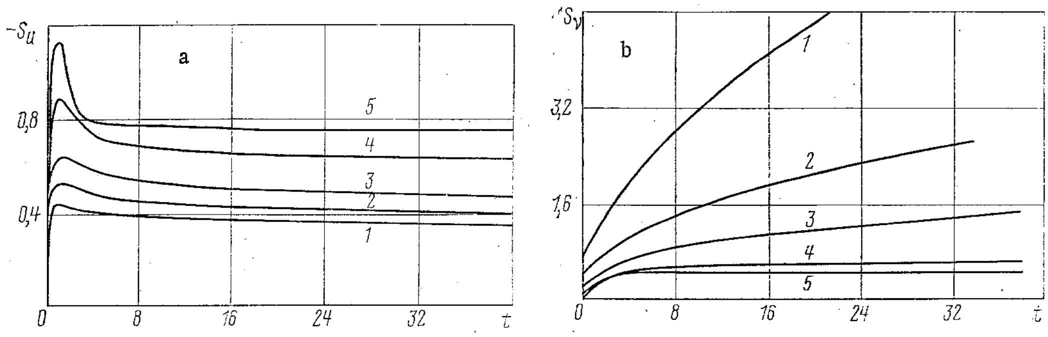

Fig. 2. Evolution of the asymmetry of arbitrary velocity fluctuations S_u (a) and of the dimensionless "viscous" moment S_ν (b) in time, at various values of the Reynolds number N_{Re}: 1) N_{Re} = 5; 2) 10; 3) 20; 4) 100; 5) 800.

experiments. Equations closed through statistical hypotheses (they have been analyzed in [18]) generally do not contain empirical constants. Although these latter equations are more accurate than equations with closure through phenomenological hypotheses (when the entire range of wave numbers is considered), their numerical solution for large values of $N_{Re,\lambda}$ is difficult. For illustration, in Fig. 1 are shown the results of numerical simulation of the function $F_u(N_{Re,\lambda})$ on the basis of one such model, viz., the Freitschnan "test field" model, covering the $36.9 \leqslant N_{Re,\lambda} \leqslant 61.5$ range [19]. These results cannot be accepted as satisfactory, because of their wide deviation from experimental data and also because of the unsatisfactory "performance" of this model, according to the authors of [19], in the intermediate range of $N_{Re,\lambda}$.

Considering these intricacies of numerical simulation by means of statistical models, we dwelled on the solution of Eq. (3) for the energy spectrum with closure through the Heisenberg hypothesis (this solution was already solved numerically before [20])

$$T(k,\ \tau) = 2\,\alpha\left[E^{1/2}(k,\ \tau)\int_0^\infty k^2E(k,\ \tau)\,dk/k^{3/2} - k^2E(k,\ \tau)\int_0^\infty (E^{1/2}(k,\ \tau)/k^{3/2})\,dk\right].$$

The constant α is related to the constant c_1 through the five-thirds power law according to the equality

$$c_1 = (4/3^{4/3})\,\alpha^{-2/3}.$$

Since c_1 = 1.4, according to experimental data [17], we obtain 0.537 for α. With the introduction of dimensionless quantities

$$K = k\lambda_u(0),\ \ t = \tau\langle u_1^2\rangle(0),$$

$$\Phi(K,\ t) = 1/4\ \pi k^2\langle u_1^2\rangle(0)\ \lambda_u^3(0),$$

$$R = \langle u_1^2\rangle^{1/2}(0)\ \lambda_u(0)/\nu$$

and a change to the new variables $\xi = \ln\,(10K)$, $\eta = 0.01tR$, $\tilde{\Phi}(\xi,\ \eta) = 0.1\Phi(K,\ t)$, Eq. (3) becomes

$$(\partial/\partial\eta + 2\,e^{\xi}/N_{Re}^2)\,\tilde{\Phi}(\xi,\ \eta) = (4\,\pi/R)\,\alpha\left[(\tilde{\Phi}^{1/2}/e^{5\xi/2})\int_{-\infty}^{\xi} e^{5\xi}\tilde{\Phi}d\xi - e^{2\xi}\tilde{\Phi}(\xi,\ \eta)\int_{\xi}^{\infty}\tilde{\Phi}^{1/2}(\xi,\ \eta)\,e^{\xi/2}\,d\xi\right]. \qquad (7)$$

Equation (7) was solved numerically, with the initial energy spectrum given in the form $E(k, 0) \sim k^2e^{-k^2}$ characteristic of undeveloped turbulence. Integration was performed by the implicit method, stable beyond the dependence on the choice of time step (a change to variables ξ and η resulted in a nonuniform k step, and the time step was selected depending on N_{Re}). The integrals on the right-hand side of Eq. (7) were evaluated according to the trapezoidal rule with a step $\Delta\xi = \ln\ 1.2$, its upper limit having been selected so as to "encompass" the dissipation range. The derivative on the left-hand side of Eq. (7) was approximated at a step $\Delta\eta = 0.0002N_{Re}(0)$. The thus calculated evolution of the inertial moment S_u and the "viscous" dimensionless parameter S_ν, the latter expressible through the spectral function as

$$S_\nu = (3\sqrt{30}/7)\int_0^\infty E(k,\ \tau)\,k^4dk\Big/\left[\int_0^\infty E(k,\ \tau)\,k^2\,dk\right]^{3/2},$$

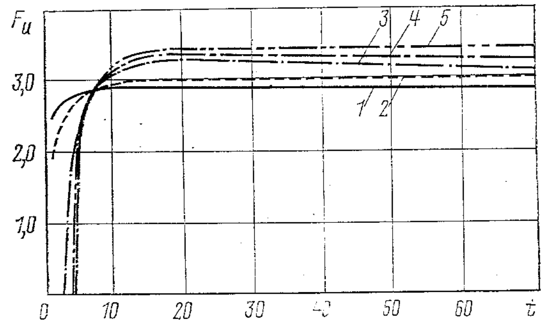

Fig. 3. Evolution of the interaction function F_u in time, at various values of the Reynolds number N_{Re}: 1) N_{Re} = 5; 2) 10; 3) 50; 4) 150; 5) 800.

is shown in Fig. 2 for various initial values of the Reynolds number N_{Re}. The evolution of function F_u, calculated from S_u and S_v, is shown in Fig. 3 and it appears here that the equilibrium mode of degeneracy stabilizes at t = 10-20, depending on the value of N_{Re}. The dependence of the thus determined equilibrium value of F_u on $N_{Re,\lambda}$ is shown in Fig. 1. A comparison of the function $F_u(N_{Re,\lambda})$ with the proposed approximations [7-9] reveals that, as $N_{Re,\lambda}$ becomes smaller, numerical simulation results in faster departure from the state of strong turbulence (an analogous behavior reflected by the approximation [10]) and, at the same time, a slower approachment to the state of weak turbulence (which may quite possibly be caused by an inadequacy of the Heisenberg model in the intermediate range of the Reynolds number). Unfortunately, the lack of reliable experimental data for the range of small and intermediate $N_{Re,\lambda}$ values makes it impossible to conclude with sufficient confidence about the behavior of the function $F_u(N_{Re,\lambda})$ in the range of weak turbulence. The only way to check the adequacy of such simulation of the function $F_u(N_{Re,\lambda})$ would probably be the method of direct numerical simulation of degenerating homogeneous and isotropic turbulence [16], which is feasible exactly in the range of weak turbulence ($0 < N_{Re,\lambda} \lessdot 35$). As an alternative consistent with experimental data one can use a simple approximation of $F_u(N_{Re,\lambda})$ in the form

$$F_u(N_{Re,\lambda}) = a_u - b_u/(1 + \beta_u (N^2_{Re,\lambda}), \tag{8}$$

where the constants a_u = 11/3 and b_u = 13/15 are found from the asymptotic estimates (5) and the constant β_u must be determined from the requirement of closest agreement with experimental data in the intermediate range of $N_{Re,\lambda}$. Since at this time there are not sufficient experimental data for intermediate $N_{Re,\lambda}$ values available, a stipulation of a numerical value for β_u may be somewhat arbitrary. Nevertheless, according to the graph in Fig. 1, the approximation (8) with β_u = 1/150 is entirely adequate for a satisfactory simulation of a degenerating homogeneous and isotropic velocity field at any value of $N_{Re,\lambda}$, i.e., in any stage of degeneracy.

NOTATION

$\overline{q^2} \equiv \overline{u_i^2}$, double the kinetic turbulence energy; $\lambda_u^2 = 5\nu\overline{q^2}/\varepsilon_u$, Taylor turbulence scale squared; $\varepsilon_u = \nu\langle(\partial u_i/\partial x_k)^2\rangle$, kinetic-energy dissipation function; and $N_{Re,\lambda} = \sqrt{\overline{q^2}}\lambda_u/\nu$, Reynolds turbulence number.

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FORMATION OF A DEVELOPED TURBULENT FLOW IN A QUADRATIC CHANNEL

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We present results of experimental investigations of aerodynamic and statistical characteristics of the region of natural transition and fully developed turbulent flow in quadratic channels.

An important place in the study of flows in pipes and channels has been the problem of the length of the entry segment in which the formation of the hydrodynamically stable state takes place. The practical application of the entry segment is most important in shorter pipes. The study of the entry segment of a turbulent flow is also important for the general understanding of the mechanism of turbulence formation.

For a long time, conflicting views were held about the length of the entry segment in pipes and channels during the turbulent flow. It was assumed here that the transition from the laminar to turbulent flow is practically sudden [1-3]. Principally, new results were obtained by Rott who showed that in the case of a minimum Reynolds numbers, the pipe contains an extended transition region with an intermittent flow regime [4]. The intermittence of the flow was explained by the fact that in the entry segment of the pipe, turbulent locks appears

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